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1991 J. Phys. A: Math. Gen. 24 L1193

(http://iopscience.iop.org/0305-4470/24/19/011)

J. Phys. A: Math. Gen. 24 (1991) L1193-L1200. Printed in the UK

LETTER TO THE EDITOR

Kinetic roughening with power-law waiting time distribution

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Received 8 August 1991

Abstract. We introduce a surface growth model where the elementary events are characterized by a waiting time distribution $P(\tau)$. Exact relations to directed polymer statistics and to continuous time random walk problems are established. For $P(\tau) \sim 1/\tau^{\mu+1}$ the behaviour is similar to that of the Zhang model where rare-event-dominated kinetic roughening occurs due to a power-law noise in the surface increments. A careful correction to scaling analysis of our numerical results in 1+1 dimensions indicates universality with the Zhang model for fixed values of μ.

Surface growth often leads to kinetic roughening [1] which can be characterized by the dynamic scaling of the correlation function $C(\boldsymbol{r}, t) = \langle |h(\boldsymbol{r}+\boldsymbol{r}', t) - h(\boldsymbol{r}', t)|^2 \rangle$ where h is the height variable of the surface measured from a d-dimensional flat substrate of linear size L. The scaling involves two exponents:

$$C(\boldsymbol{r}, t) \sim |\boldsymbol{r}|^{2\zeta} f(t/|\boldsymbol{r}|^{\zeta/\beta}) \tag{1}$$

where ζ is the roughness exponent and $\zeta/\beta = z$ is the dynamic exponent. The scaling function f has the properties $f(a) \to$ constant for $a \gg 1$ and $f(a) \to a^{2\beta}$ for $a \ll 1$ in a very large system ($L \gg t^{1/z}$).

For a large class of systems an adequate theory of kinetic roughening was presented by Kardar *et al* [2] who set up the following Langevin-type equation for the time evolution of h:

$$\partial_t h = \nu \nabla^2 h + (\lambda/2)(\nabla h)^2 + \eta \tag{2}$$

with ν and λ being phenomenological parameters. The term η represents noise which is usually supposed to be uncorrelated and bounded (e.g. Gaussian). For this case equation (2) leads to the exact exponents $\zeta = \frac{1}{2}$ and $\beta = \frac{1}{3}$ for substrate dimension 1 (i.e. in 1+1 dimensions). Furthermore, there is a relationship between the two exponents [3]

$$\zeta + \zeta/\beta = 2 \tag{3}$$

which holds in any dimension. In higher dimensions extensive numerical investigations have been carried out either based on the direct solution of (2) [4, 5] or on simulating models [6-9] which are expected to be described by the KPZ theory. Thus we know the values of the exponents to a high accuracy.

0305-4470/91/191193+08$03.50

In some recent experiments [10–12] on quasi-(1+1)-dimensional surface growth exponents were obtained which are significantly larger than the values predicted by the KPZ theory. As a possible solution of this discrepancy Zhang proposed [13] that a power-law distribution of the noise amplitudes

$$P(\eta)=\begin{cases}\dfrac{\mu}{\eta^{\mu+1}} & \text{for } \eta>1\\ 0 & \text{otherwise}\end{cases} \tag{4}$$

(instead of a bounded distribution) could lead to larger exponents. In fact, a great deal of numerical evidence has accumulated [13–17] and supports this idea.

A simple scaling argument [18, 19], based on the assumption that the rare events determine the roughness, leads to the following dependence of the exponents ζ and β on μ:

$$\zeta(\mu)=\frac{d+2}{\mu+1} \qquad \text{and} \qquad \beta(\mu)=\frac{d+2}{2\mu-d} \tag{5}$$

where the relationship (3) is still valid. As long as the exponents (5) are larger than those without power-law noise the rare events are relevant. This way (5) predicts a critical value μ_c above which no influence of the power-law noise on the exponents should occur; e.g. $\mu_c=5$ in $1+1$ dimensions. The numerical results in this respect are controversial: Amar and Family [14] detected μ-dependent exponents up to $\mu=7$–8 while Buldyrev *et al* [15] concluded that μ_c should in fact be close to 5.

The analogue of rare-event-governed roughening could also be observed in the statistics of directed polymers in random media [20] since this problem is related to kinetic roughening as already noticed by Kardar *et al* [2]. In a recent paper [21] this relationship was made more direct by relating Eden growth and polymer configurations via a waiting time distribution of the growth events. The crucial point was the identification of the waiting times of the growth problem with the random energy values of the sites for the directed polymer.

In this letter we follow the lines of Roux *et al* [21]. However, there are two essential differences: (i) in contrast to Roux *et al* our mapping of the growth problem to the statistics of directed polymers is *exact*; (ii) for our model a power-law distribution of the waiting times leads to rare-event-governed growth which was not the case for the Eden model considered by Roux *et al*. The reason for this latter difference is that we take a model with negative effective λ (in (2)) as the starting point of our considerations while the Eden model is characterized by a positive λ. Our approach relates the Zhang model to biased continuous time random walks in a lattice gas. Numerical simulation of the model is used to clarify questions of universality.

The model we introduce is a reformulation and extension of the single-step model [3, 22] on a square lattice tilted by 45°. Each lattice site $\boldsymbol{R}=n_1\boldsymbol{a}_1+n_2\boldsymbol{a}_2$ is assigned a substrate coordinate $x(\boldsymbol{R})=n_1-n_2$ and a height $h(\boldsymbol{R})=n_1+n_2$, where $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ are edge vectors pointing at 45° and 135°, respectively. In a strip geometry, a surface of L lattice points $x=1,\ldots,L$ satisfies the 'single-step rule' $|h(x+1)-h(x)|=1$ and is closed under periodic boundary conditions in the horizontal direction. The starting configuration at $t=0$ is a crystal which has a 'flat' surface at $t=0$, with h equal to 0 and -1 for even and odd integers x. Growth $(h(x)\to h(x)+2)$ can take place only at local minima of the height $h(x)$. Following Roux *et al*, we formulate the growth model in terms of waiting times $\tau(\boldsymbol{R})$ which are independent stochastic variables drawn from a given distribution $P(\tau)$. Each unoccupied site $\boldsymbol{R}$ at $h>0$ is assigned a waiting time

$\tau(\boldsymbol{R}) > 0$. The waiting time at $\boldsymbol{R}$ is counted from the moment when this site becomes a growth site, i.e. once the two lower neighbours at $\boldsymbol{R}-\boldsymbol{a}_1$ and $\boldsymbol{R}-\boldsymbol{a}_2$ are occupied. When the waiting period expires, this site gets occupied. We first show how this model is mapped onto the problem of directed polymers.

The site $\boldsymbol{R}$ becomes occupied at a time $t(\boldsymbol{R})$ which is obviously given by the iteration

$$t(\boldsymbol{R}) = \tau(\boldsymbol{R}) + \max\{t(\boldsymbol{R}-\boldsymbol{a}_1),\, t(\boldsymbol{R}-\boldsymbol{a}_2)\}. \tag{6}$$

This is in contrast to the Eden model where, as occupation of any one of its neighbouring sites makes $\boldsymbol{R}$ a growth site, the max-rule in (6) is replaced by a min-rule. Iterating (6) yields

$$t(\boldsymbol{R}) = \max_{S}\left\{\sum_{\boldsymbol{R}'\in S} \tau(\boldsymbol{R}')\right\} \tag{7}$$

where each path S is directed upwards along the edges of the squares, starting from any site at $h=1$ and terminating in $\boldsymbol{R}$. Identifying $-\tau(\boldsymbol{R})$ with the site energy at $\boldsymbol{R}$, the problem posed by (7) is identical to finding a ground state configuration of a polymer directed along the diagonal on the square lattice with a random site-energy distribution. Thus we have established an *exact* mapping between the waiting-time single-step model and the directed polymer problem at zero temperature. The growth process, like the transfer matrix approach, *naturally* follows minimum energy paths of the directed polymer problem. This is in contrast to the case studied by Roux *et al* [21] because the overhangs in the Eden model lead to the violation of the directedness.

Denoting the ground state energy by E, in our mapping we identify $-E$ with the time of growth and the length of the polymer l with the height h of the surface. Similar to the Eden model considered by Roux *et al*, the exponent β which describes the power-law increase of height fluctuations as a function of t, and which occurs in the finite time correction of the height [23],

$$\langle h\rangle \simeq vt + h_1 t^{\beta} \tag{8}$$

is thus identified with the exponent ω which describes the non-analytic correction to the energy E as a function of the length of the polymer l,

$$\langle E\rangle \simeq e_0 l + e_1 l^{\omega}. \tag{9}$$

Here and elsewhere $\langle\ldots\rangle$ denotes the average over the waiting time or random energy distribution. The equivalence of (8) and (9) also yields the following relations between the constants,

$$v = -1/e_0 \qquad h_1 = e_1 v^{1+\beta}. \tag{10}$$

Since the minimum energy is approached from above for increasing l, e_1 and hence h_1 are expected to be positive. In contrast, the same argument applied to the Eden model yields a negative h_1. As $h_1 \propto -\lambda$ [23] this difference in the correction to the steady-state growth velocity v is consistent with the notion that Eden growth and single-step models correspond to positive and negative values of λ in the KPZ equation, respectively, which was established previously on phenomenonological grounds.

A second exponent in the growth problem is z which describes the power-law increase of the lateral correlation length $\xi \sim t^{1/z}$. In the work of Roux *et al* [21], $1/z$ was identified with the roughness exponent ν' of a directed polymer, $x \sim l^{\nu'}$, through a finite-size scaling analysis. Here we show that the relation can be established also by considering an infinite system, $L \to \infty$. The mapping discussed above applies to *any*

site on the surface. In a given realization of the growth process, however, the waiting times and thus the random site-energies are fixed. The minimum energy paths to surface sites sufficiently close to each other may overlap significantly, thus becoming correlated. Identifying the lateral correlation length with the typical transverse meandering distance of the polymer, one obtains $\nu' = 1/z$.

Let us turn to the question of how our model is related to other surface growth models. In the single-step model considered in [3, 22], the surface moves via random deposition at its local minima. This model can be extended to include noise reduction [24] such that growth takes place after a site has accumulated $M \geqslant 1$ hits. For a sufficiently large system, the above scheme is equivalent to our model with a continuous waiting-time distribution

$$P(\tau) = \frac{\tau^{M-1}}{\Gamma(M)} e^{-\tau} \tag{11}$$

where the $M = 1$ case was discussed by Roux *et al* [21].

Growth models with waiting time distributions can be related also to models with two sublattices and parallel updating where a growth site at a given time becomes occupied with a probability p. For this purpose we restrict $\tau(\boldsymbol{R})$ to half-integer values, and do the count-down in units of $\frac{1}{2}$. The two sublattices correspond to sites at even and odd values of x. As the growth sites at $t = 0$ belong to the even sublattice, growth at sites with odd (even) x values happens only at integer (half-integer) times. Then we obtain the following probability distribution of waiting times,

$$P(\tau) = p(1-p)^{\tau - 1/2} \qquad \tau = \tfrac{1}{2}, \tfrac{3}{2}, \ldots . \tag{12}$$

It is straightforward to see the generalization of the waiting-time description and hence the mapping to directed polymers problem to the hypercube-stacking model [8] without evaporation and to the restricted solid-on-solid growth models of Kim and Kosterlitz [7] in any dimension, which we shall not elaborate here.

It is well known that the $(d+1)$-dimensional single-step model can be mapped onto a d-dimensional spin model [3, 22]. The mapping is particularly simple in $1+1$ dimensions: the surface configuration is then equivalent to a state of a one-dimensional Ising system or a lattice gas if spins s_i are defined as $s_i = h_i - h_{i-1}$, where $s_i = 1$ (-1) corresponds to an occupied (empty) site in the lattice gas picture. A local surface minimum is represented by a pair of neighbouring sites on the chain with the left one empty and right one occupied. Growth at the minimum is realized by an exchange of the two sites, i.e., particle hopping to the left.

Plischke *et al* [22] generalized this model by allowing evaporation at the 'peaks' of the surface i.e. allowing particles to hop to their empty neighbours on the right. There are rates defined for right and left hopping thus the model corresponds to a diffusion problem in a lattice gas. Such a model can also be specified in terms of waiting times: whenever a site becomes a candidate for growth (evaporation) a waiting time is generated. Either the growth (evaporation) event happens after the waiting period expires or the site is blocked because of a change in the neighbourhood, in which case the waiting time is reset.

Assuming a continuous distribution for the waiting times, the model corresponds to a continuous time random walk problem in a lattice gas.

In the remainder of the letter we shall focus on the single-step model with the following distribution of waiting times for growth,

$$P(\tau) = (\tau + \tfrac{1}{2})^{-\mu} - (\tau + \tfrac{3}{2})^{-\mu} \qquad \tau = \tfrac{1}{2}, \tfrac{3}{2}, \tfrac{5}{2} \ldots . \tag{13}$$

No evaporation is allowed. At large τ, $P(\tau)$ falls off as $\mu\tau^{-\mu-1}$. In contrast to the Eden model where the time is mapped to the energy (rather than the negative energy) of a polymer, we expect the power-law waiting time distribution to affect surface roughness here.

We have performed simulations of the single-step model without evaporation with the distribution (13) to check the predicted values of the exponents as a function of μ. For the data presented below the system size was chosen at $L = 10^5$ surface sites and averages were performed over 20–40 realizations. Each data set-up to $t = 1024$ took about 200 minutes on an IBM 3090 machine without vectorization facility. Figure 1 shows the minimum energy paths (or ancestry tree) to a small part of the surface in a typical run at $\mu = 3$. The growth of lateral correlations is evident from the expansion of the size of the trees.

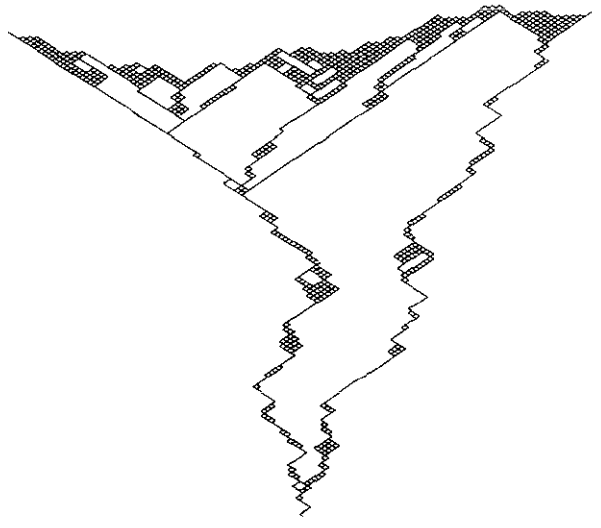

Figure 1. Minimum energy configurations to part of the surface ($\mu = 3$).

We have measured the mean-square surface width $w^2(t) = \langle \overline{h^2}(t) - \bar{h}^2(t) \rangle$ and the height-height correlation function $C(r, t)$ at $t = 2^m$, $m = 0, 1, \ldots, 10$. (The quantities with a bar are spatial averages.) Figure 2 shows the $\Delta w^2(t) \equiv w^2(2t) - w^2(t)$ against t on a log-log scale for $\mu = 2.5, 4$ and 7.

In the usual single-step model the dominant correction to scaling is the constant intrinsic width [25]. The quantity Δw^2 does not contain this correction anymore [8]. Similarly to other investigations [15–17] of rare-event-governed roughening we see strong corrections to scaling in addition to that caused by the intrinsic width (figure 2). The lack of a good understanding of the correction-to-scaling term prevented us from obtaining accurate estimates of β.

In contrast, we show how the height-height correlation data can be extrapolated to $t = \infty$. We simulated large ($L = 10^5$) systems for rather short times so that corrections mainly stem from finite time and not from finite size. Our extrapolation scheme is based on a dynamic scaling ansatz for the height-height correlation function in momentum space,

$$\langle h(k, t)h(k', t) \rangle \simeq k^{-1-2\zeta}[\tilde{f}_0 + \tilde{f}(tk^z)]\delta(k + k') \tag{14}$$

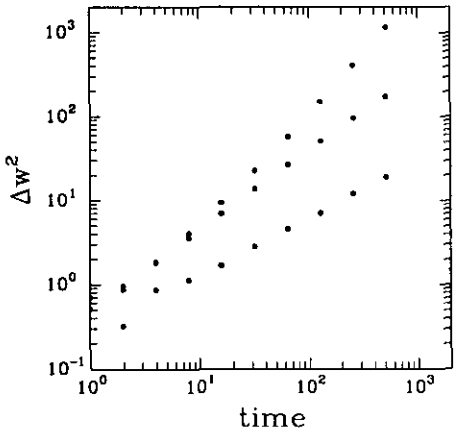

Figure 2. Corrected mean-square surface width data against time. $\mu = 2.5$, 4 and 8 from top to the bottom. The system size was $L = 10^5$ and averages over 40 runs were taken.

where $f_0 + \tilde{f}(\tilde{\rho}) \sim \tilde{\rho}^{2\beta}$ for $\tilde{\rho} \ll 1$ and $\tilde{f}(\tilde{\rho}) \to 0$ for $\tilde{\rho} \gg 1$. Fourier transforming the above expression yields

$$C(r, t) \simeq \frac{2}{\pi} \int_{2\pi/L}^{\pi/a} \mathrm{d}k (1 - \cos kr) k^{-1-2\zeta} [\tilde{f}_0 + \tilde{f}(tk^z)] \tag{15}$$

where a is the lattice constant. The term proportional to $\tilde{f}_0$ yields the asymptotic time independent correlation function to which we want to extrapolate our time dependent data. Assuming that the function $\tilde{f}(tk^z)$ decays quickly to zero for increasing arguments the cosine can be expanded. Up to fourth order we obtain

$$C(r, t) = C(r, \infty) + f_1 r^2 t^{-2(1-\zeta)/z} + f_2 r^4 t^{-2} \tag{16}$$

for a fixed r. The parameters $C(r, \infty)$, f_1 and f_2 are determined by a least-squares fit for fixed r. The exponent ζ is an input into this fit and is calculated self-consistently such that it agrees with the slope of the log–log plot of $C(r, \infty)$. Actually the fit is not very sensitive to the choice of ζ. For $r \geqslant 10$ the dependence of the fitted values for f_1 on r is only on the second or third significant digit, while f_2 is at least one order of magnitude smaller than f_1 in most cases. Figure 3 shows the extrapolated correlation function for different values of μ. Figure 4 presents the consecutive slopes for $\mu = 3$ and 7 from figure 3 against $1/r$. At the largest r-values the effective exponents drop significantly which we take as an indication of the saturation due to the short times used in the simulations. Ignoring these parts of the curves, we extrapolate the exponent ζ. The obtained values are given in figure 5.

In this letter we have shown how to generalize the model of rare-event-governed roughening to the case where λ is negative in (2). For this purpose we introduced waiting times into the single-step model and assumed a power-law distribution for them (13). Our model can be mapped exactly to the directed polymer problem and to continuous time random walks of interacting particles. Due to a careful correction to scaling analysis we were able to obtain accurate results for the exponent ζ from short time runs for correlation function. The effective exponents lie above the values of (5) up to $\mu = 7$. If our estimates are close to the true values, one has to conclude that (5) should be interpreted as lower bounds for the exponents. These findings are similar

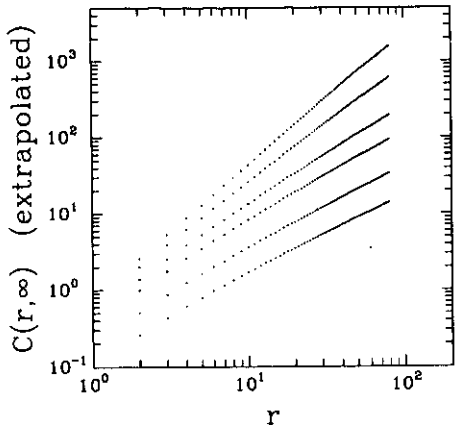

Figure 3. The height-height correlation data extrapolated to $t \to \infty$. From top to bottom $\mu = 2.5, 3, 4, 5, 7, 9$. The data are obtained for short times and therefore indication of saturation can be seen.

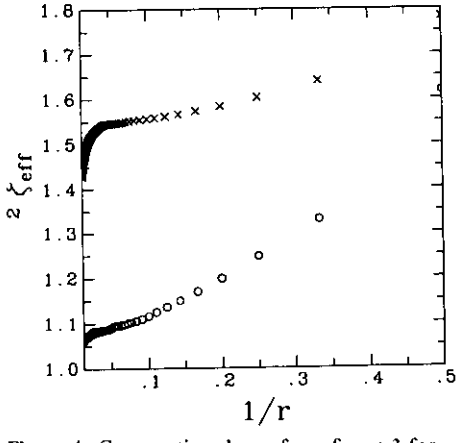

Figure 4. Consecutive slopes from figure 3 for $\mu = 3$ (×) and 7 (○).

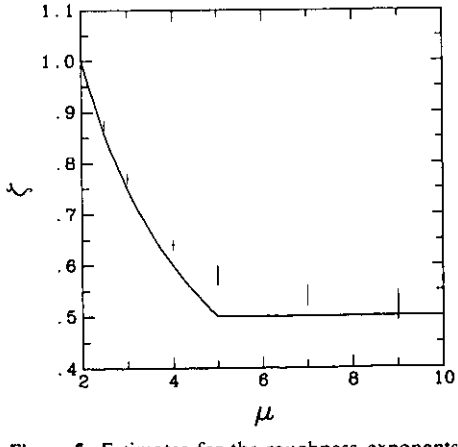

Figure 5. Estimates for the roughness exponents ζ. The solid line indicates the predicted values according to (5).

to the ones by Amar and Family [14] who simulated the Zhang model with parallel updating and a positive λ.

This research was supported in part by the DFG through SFB 237 and 341. JK thanks the Humboldt foundation for support. Thanks are due to the GMD at St Augustin for computer time on the Alliant parallel computer.

Note added. After submission of this letter we learned of an independent study of a similar model by M H Jensen and I Procaccia.

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